1
To Infinity and Beyond

In this unbelievable universe in which we live there are no absolutes. Even parallel lines, reaching into infinity, meet somewhere yonder.

Pearl S. Buck, *A Bridge for Passing*

The infinite is a concept so remarkable, so strange, that contemplating it has driven at least two great mathematicians over the edge into insanity.

In the *Hitch-hiker's Guide to the Galaxy*, Douglas Adams described how the writers of his imaginary guidebook got carried away in devising its introduction:

> 'Space,' it says, 'is big. Really big. You just won't believe how vastly, hugely, mind-bogglingly big it is. I mean, you may think it's a long way down the street to the chemist, but that's just peanuts to space. Listen . . .' and so on. After a while the style settles down a bit and it starts telling you things you actually need to know . . . [1]

Infinity makes space seem small.

Yet this apparently unmanageable concept is also with us every day. My daughters were no older than six when they first began to count quicker and quicker, ending with a blur of words and a triumphant cry of 'infinity!' And though infinity may in truth make space seem small, when we try to think of something as vast as the universe, infinite is about the best label our minds can apply.

Anyone who has broken through the bounds of basic

created with an unlicensed copy of nitro PDF express upgrade: www.nitropdf.com/express

mathematics will have found the little ∞ symbol creeping into their work (though we will discover that this drunken number eight that has fallen into the gutter is not the real infinity, but a ghostly impostor). Physicists, with a carelessness that would make any mathematician wince, are cavalier with the concept. When I was studying physics in my last years at school, a common saying was 'the toast rack is at infinity'. This referred to a nearby building, part of Manchester Catering College, built in the shape of a giant toast rack. (The resemblance is intentional, a rare example of humour in architecture. The companion building across the road, when seen from the air, looks like a fried egg.) We used the bricks on this imaginative structure to focus optical instruments. What we really meant by infinity was that the building was 'far enough away to pretend that it is infinitely distant'.

Infinity fascinates because it gives us the opportunity to think beyond our everyday concerns, beyond *everything* to something more – as a subject it is quite literally mind-stretching. As soon as infinity enters the stage, it seems as if common sense leaves. Here is a quantity that turns arithmetic on its head, making it seem entirely feasible that $1 = 0$. Here is a quantity that enables us to cram as many extra guests as we like into an already full hotel. Most bizarrely of all, it is quite easy to show that there must be something that is bigger than infinity – which surely should be the biggest thing there could possibly be.

Although there is no science more abstract than mathematics, when it comes to infinity, it has proved hard to keep spiritual considerations out of the equation. When human beings contemplate the infinite, it is almost impossible to avoid things theological, whether in an attempt to disprove or prove the existence of something more, something greater than the physical universe. Infinity has this strange ability to be many things at once. It is both practical and mysterious. Mathematicians, scientists and engineers use it quite happily because it works – but they consider it a black box, having the same relationship with it that most of us

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express

do with a computer or a mobile phone, something that does the job even though we don't quite understand how.

The position of mathematicians is rather different. For them, modern considerations of infinity shake up the comfortable, traditional world in the same way that physicists suffered after quantum mechanics shattered the neat, classical view of the way the world operated. Reluctant scientists have found themselves having to handle such concepts as particles travelling backwards in time, or being in two opposite states at the same time. As human beings, they don't understand why things should be like this, but as scientists they know that if they accept the picture it helps predict what actually happens. As the great twentieth-century physicist Richard Feynman said in a lecture to a non-technical audience:

> It is my task to convince you not to turn away because you don't understand it. You see, my physics students don't understand it either. That is because I don't understand it. Nobody does.[2]

Infinity provides a similar tantalizing mix of the normal and the counter-intuitive.

All of this makes infinity a fascinating, elusive topic. It can be like a deer, spotted in the depths of a thick wood. You will catch a glimpse of beauty that stops you in your tracks, but moments later you are not sure if you saw anything at all. Then, quite unexpectedly, the magnificent animal stalks out into full view for a few, fleeting seconds.

A real problem with infinity has always been getting through the dense undergrowth of symbols and jargon that mathematicians throw up. The jargon is there for a very good reason. It's not practical to handle the subject without some use of these near-magical incantations. But it is very possible to make them transparent enough that they don't get in the way. We may then open up clear views on this most remarkable of mathematical creatures – a concept that goes far beyond sheer numbers, forcing us to question our understanding of reality.

Welcome to the world of infinity.

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express

2
Counting on Your Fingers

Alexander wept when he heard from Anaxarchus that there was an infinite number of worlds; and his friends asking him if any accident had befallen him, he returns this answer:

'Do you not think it a matter worthy of lamentation that when there is such a vast multitude of them, we have not yet conquered one?'

Plutarch, *On the Tranquillity of the Mind*

COUNTING A sequence of numbers, one after the other, is a practice that is ingrained in us from childhood. The simple, step-by-step progression of the numerals is so strong that it can be surprisingly difficult to break out of the sequence. Try counting from one to ten as quickly as you can out loud in French (or another language where you know the basics, but aren't particularly fluent). Now try to keep up the same speed counting back down from ten to one. The result is usually hesitation; the rhythm breaks as we fumble around for the next number. We are tripped up trying to untangle that deep-seated progression.

Number sequences have become embedded in our culture, often as the focus of childhood rhymes. The most basic of these are simple memory aids dating back to our first attempts to count:

One, two, buckle my shoe,
Three, four, knock on the door,

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express

Five, six, pick up sticks,
Seven, eight, lay them straight,
Nine, ten, a big, fat hen,
Eleven, twelve, dig and delve,
Thirteen, fourteen, maids a'courting,
Fifteen, sixteen, maids in the kitchen,
Seventeen, eighteen, maids in waiting,
Nineteen, twenty, my plate's empty.

There's a degree of desperation in some of the later rhymes in the sequence, but also a fascinating reminder of a world that has disappeared in its imagery of shoe buckles and maids. The repetitious, hypnotic rhythm helps drive the number values into place.

Other doggerel is more oriented to singing than the basic chanted repetition of 'one, two, buckle my shoe', a typical example being

One, two, three, four, five,
Once I caught a fish alive,
Six, seven, eight, nine, ten,
Then I let it go again.

Equally valuable for practice with counting are songs like *Ten Green Bottles*, running downwards to make a child's grasp of the numbers more flexible.

But number rhymes aren't limited to helping us learn the basics of counting. More sophisticated verses add symbolism to sequence. It's difficult not to feel the power of numbers coming through in a rhyme like the magpie augury. This traditional verse form, familiar to UK children's TV viewers of the 1970s from its use in the theme song of the magazine programme *Magpie*, links the number of magpies (or occasionally crows) seen at one time with a prediction of the future. It's not so much about counting as about fortune-telling.

The TV show used the first part of a common sanitized version:

One for sorrow, two for joy,
Three for a girl and four for a boy,

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express

Five for silver, six for gold,
Seven for a secret, never to be told,
Eight for a wish and nine for a kiss,
Ten for a marriage never to be old.[3]

But there's more earthy realism in this early Lancashire variant:

One for anger, two for mirth,
Three for a wedding and four for a birth,
Five for rich, six for poor,
Seven for a bitch [or witch], eight for a whore
Nine for a burying, ten for a dance,
Eleven for England, twelve for France.[4]

Many children develop a fascination with the basic sequence of counting numbers. Once youngsters have taken on board the rules for naming the numbers, it's not uncommon for their parents to have to beg them to stop as they spend an inordinate amount of time counting up and up. Perhaps their intention is to get to the end, to name the 'biggest number'. But this is a task where a sense of completeness is never going to be achieved. A child could count for the rest of his or her life and there would still be as many numbers to go. It seems that children are fascinated by the order, the simple pattern of such a basic, step-by-step string.

It is part of human nature to like order, to see patterns even where no patterns exist. When we look at the stars, we imagine that they form constellations – shapes that link these points of light into a skeletal picture – where in reality there is no link between them. You only have to consider the Centaurus constellation in the southern sky. Its brightest star, Alpha Centauri, is the nearest to ours, a mere four light years distant; the next brightest in the constellation, Beta Centauri (or Agena), lies 190 light years away, more than 45 times more distant. We are mistakenly linking together two objects that are separated by around 1,797,552,000,000,000 kilometres.

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express

Our own Sun is much closer to Alpha Centauri than Agena is, yet we would hardly think of the Sun and Alpha Centauri as forming a pattern. Alpha and Beta Centauri are no more connected than Houston and Cairo are simply because they lie near the same latitude. Our eyes and brains, looking for structure in the myriad of winking points in the sky, deceive us into finding patterns.

We look for patterns primarily to aid recognition. Our brains simplify the complex shapes of a predator or of another human face into patterns to enable us to cope with seeing them from different directions and distances. In the same way that we look for patterns in the physical objects around us, we appreciate patterns in numbers, and, of these, few are simpler and more easily grasped than the series of whole or counting numbers 1, 2, 3, 4, 5, 6, . . .

The ellipsis at the end of the sequence, that collection of three dots '. . .' is a shorthand that stretches beyond mathematics, though we have to be a little careful about how it is being used here. In normal usage it simply means 'and so on' for more of the same, but mathematicians, more fussy than the rest of us, take it specifically to mean 'and so on without *any* end'. There is no point at which you can say the sequence has stopped, it just goes on. And on. And on.

From the earliest days of pre-scientific exploration of both the natural world and the world of the mind, such chains of numbers were examined with fascination. They are prime inhabitants of the landscape of mathematics, as rich and diverse as any family of animals in the biological terrain. Some sequences are almost as simple as the basic counting numbers, for example doubling the previous number in the sequence to produce

$$1,\ 2,\ 4,\ 8,\ 16,\ 32,\ 64,\ 128, \ldots$$

But things don't have to be so neatly ordered. You can have sequences that vary in direction, building in a dance-like move-

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express

ment, alternating two steps forward and one step back:

$$1,\ 3,\ 2,\ 4,\ 3,\ 5,\ 4,\ 6,\ 5,\ 7,\ldots$$

Or each value can be the sum of the previous two, the so-called Fibonacci numbers:

$$0,\ 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ldots$$

Moreover, we can move on from addition and subtraction to wilder excursions that multiply, taking off like a flock of birds surprised on a lake: the squares, the original counting numbers multiplied by themselves, for instance,

$$1,\ 4,\ 9,\ 16,\ 25,\ 36,\ 49,\ldots,$$

or the fast-accelerating progress of a sequence that multiplies the two previous entries,

$$1,\ 2,\ 2,\ 4,\ 8,\ 32,\ 256,\ 8192,\ldots$$

Most of these types of sequence were known by the Greek philosophers who first pondered the nature of numbers. But one particular class seemed particularly to have fascinated them. These were progressions, not of whole numbers, but of fractions.

The simplest fractional sequence takes each whole number and makes it the bottom part of the fraction:

$$1,\ \tfrac{1}{2},\ \tfrac{1}{3},\ \tfrac{1}{4},\ \tfrac{1}{5},\ \tfrac{1}{6},\ldots$$

This sequence of numbers is not particularly special. If we were to add each term to the next, the total would grow without limit. But the Greek philosophers noticed a very different – a bizarrely different – behaviour when they made a tiny change. Instead of using the counting numbers in the bottom part of the fraction, the new sequence is formed by doubling the bottom part of the fraction. The result,

$$1,\ \tfrac{1}{2},\ \tfrac{1}{4},\ \tfrac{1}{8},\ \tfrac{1}{16},\ \tfrac{1}{32},\ldots,$$

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express

has a very strange property, strange enough for the philosopher Zeno to use it as the basis for two of his still-famous paradoxes.

We know very little directly of Zeno's work. All his writing was lost except for a few hundred words (and even the attribution of those to him is questionable). What remains is the second-hand commentary of the likes of Plato and Aristotle, who were anything but sympathetic to Zeno's ideas. We know that Zeno was a student of Parmenides, who was born around 539 BC. Parmenides joined the settlement at Elea in southern Italy. The ruins of this Phoenician colony can still be seen outside the modern-day Italian town of Castellammare di Velia. It was here that the Eleatic school pursued a philosophy of permanent, unchanging oneness – believing that everything in the universe is as it is, and all change and motion is but illusion.

For all we know, Zeno may have contributed much to Eleatic philosophy, but now he tends to be remembered as a mathematical one-hit wonder. What has come to us, even though only as a dim reflection in the comments of others, is his fascination with tearing apart the way we think of motion. This demonstrates a fundamental belief of the Eleatics, the denial of the existence of change, but even in this indirect form it seems possible to detect a metaphorical glint in Zeno's eye as he puts up his arguments. The later writers who pass on the paradoxes point out that this was a youthful effort, and though the intention of describing them this way was to be scornful, in fact there is a very positive element of youthful challenge in these ideas.

In all, forty of Zeno's reflections on the static universe have been recorded, but it is four of them that continue to capture the imagination, and it is these four that particularly impinge on the consideration of motion and of that strange sequence of numbers $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$

The most straightforward of the four tells the story of Achilles and the tortoise. Achilles, arguably the fastest man of his day, the equivalent of a modern sports star, takes on the ponderously slow tortoise in a race. Considering the result of a rather similar race

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express

in one of Aesop's fables (roughly contemporary with Zeno's paradoxes), it's not too surprising that the tortoise wins. But unlike the outcome of the race between the tortoise and the hare, this unlikely result is not brought on by laziness and presumption. Instead it is the sheer mechanics of motion that Zeno uses to give the tortoise the winner's laurels.

Zeno assumes that Achilles is kind enough to give the tortoise an initial advantage – after all this is hardly a race of equals. He allows the tortoise to begin some considerable distance in front of him. In a frighteningly small time (Achilles is quite a runner), our athletic hero has reached the point that the tortoise started from. By now, though, however slow the tortoise walks, it has moved on a little way. It still has a lead. In an even smaller amount of time, Achilles reaches the tortoise's new position – yet that extra time has given the tortoise the opportunity to move on. And so the endless race carries on with the physical equivalent of those three dots, Achilles eternally chasing the tortoise but never quite catching it.

Another of the paradoxes, called the dichotomy, is very closely related. This shows that it should be impossible to cross a room and get out of it. Before you cross the room, you have to reach the half-way point. But you can't get there until you've reached one-quarter of the way across the room. And that can't be reached before you get to the one-eighth point – and so on. The sequence

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$$

gives an insight into what is happening. You are never even going to reach the $\frac{1}{2}$, because you haven't first reached the $\frac{1}{4}$, because you didn't get to the $\frac{1}{8}$th marker, and so on – and you can never get started because you can't define the initial point. You can run down the sequence, splitting the distance further and further, forever. Which is the first point you reach? We can't say, and so, Zeno argues, you can never achieve motion.

It's easy to dismiss these two paradoxes by pointing out that

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express

Achilles' steps do not get smaller and smaller, so, as soon as he is within a pace of the tortoise, his next stride will take him past it. The same goes for our escape from the room, but in reverse – the first step you take will encompass all of the smaller parts of the sequence up to whatever fraction of the way across the room you get to. But that misses the point of the story. Zeno, after all, was trying to show that the whole idea of motion as a continuous process that could be divided up as much as you like was untenable.

It is helpful to put Zeno's paradox of Achilles and the tortoise alongside our sequence

$$1, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{8}, \tfrac{1}{16}, \tfrac{1}{32}, \ldots$$

When we consider Achilles and the tortoise, let's make the rather rash assumption that the tortoise moves half as fast as the athlete (maybe Achilles was having a bad day, or the tortoise was on steroids). Then in the time Achilles moved a metre, the tortoise would have moved half a metre. In the time Achilles made up that half metre, the tortoise would do an extra quarter. As Achilles caught up the quarter, the tortoise would struggle on an eighth. All of a sudden, those numbers are looking very familiar. But the parallel isn't all that makes it interesting. Because once we start adding up the numbers in that series, something strange happens. Let's look at the same series of fractions, but add each new figure to the previous one to make a running total. Now we have

$$1,\ 1\tfrac{1}{2},\ 1\tfrac{3}{4},\ 1\tfrac{7}{8},\ 1\tfrac{15}{16},\ 1\tfrac{31}{32}, \ldots$$

However far you take the series you end up with 1 and something more, something more that is getting closer and closer to another 1 (making 2 in all), but never quite getting there. It doesn't matter how far you go, the result remains less than 2. Think of the biggest multiple of 2 you can possibly imagine – we'll give it a made up name, a 'thrupple'. Then by the time you reach it you

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express

will have

$$1 + \frac{\text{thrupple} - 1}{\text{thrupple}},$$

but not quite 2.

Giving a huge number an arbitrary name like 'thrupple' isn't quite as bizarre as it seems. The biggest number around with a non-compound name, the googol, doesn't just sound like a childish name, it actually was devised by a child. According to the story of its origin, American mathematician Edward Kasner was working on a blackboard at home in 1938 and for some reason had written out the number below. 'That looks like a googol,' said his nine-year-old nephew Milton Sirrota. And the name stuck. That part of the story seems unlikely – it's more credible that Kasner was simply looking for a name for a number bigger than anyone can sensibly conceive and asked young Milton for suggestions. Either way, a googol is the entirely arbitrary number:

10,000,000,000,000,000,000,000,000,000,000,000,000,
000,000,000,000,000,000,000,000,000,000,000,000,
000,000,000,000,000,000,000,000,000,000,000,000

or 1 with 100 noughts after it.

Back at the sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$, what we find, then, is that however many times we add an item to the list, we will never quite reach a total of 2. As soon as Achilles' stride breaks this apparent barrier the tortoise's lead is done for, but the series itself can't break out of its closer and closer approach to 2.

A modern equivalent of the translation of this sequence into a physical reality, an Achilles and the tortoise for the twenty-first century, would be to imagine a series of mirrors reflecting a particle of light, a photon, each mirror half the distance from the next, set in a spiral. The obvious paradox here is that however many mirrors you cram in, however many reflections you allow for, the light will only travel a limited distance. But there is another, more

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express

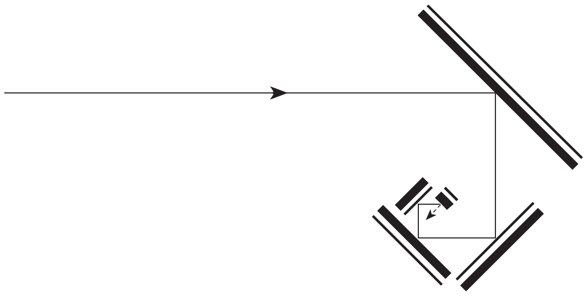

Figure 2.1 The disappearing photon.

subtle consideration. What happens to the photon at the end of the process? Where does it go? We know that after the first mirror it begins to spiral inwards, but light is incapable of stopping. The photon must continue travelling at 300,000 kilometres per second. So where does this particle go to?

In practice this seems to be one of those questions that have no meaning, because we would have to stray beyond the bounds of physical reality to reach the end result. Even if it were possible to divide up *space* into infinitely small chunks (we will return to the practicalities of this in the penultimate chapter), we know that physical matter isn't a continuous substance that can be divided forever, always coming up with a smaller version of the same thing. Reflection of light depends on an interaction between the photon of light and an electron in the material that it is being reflected off. Eventually, as the mirrors got smaller and smaller to fit into the remaining space they would have to become smaller than an atom, smaller than an electron – at which point reflection could no longer occur and the photon would continue on its journey without being further reflected.

In a moment we will return to that elusive sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$, which seems as if it should add up to 2 but never quite makes, but first, for neatness, let's finish off Zeno and his paradoxes.

The other two mental pictures Zeno painted attack the con-

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express

ventional view of motion in a different way. Perhaps most famous of all is Zeno's arrow. He describes an arrow, flying through space. After a certain amount of time has passed, it will have moved to a new position. But now let's imagine it at a particular instant in time. The arrow must be somewhere. You can imagine it hanging in space like a single frame from a movie. That's where the arrow is at, say, exactly ten minutes past two.

This is where the visual imagery of film comes in handy. There is now a video technique available that seems to make time stop. An object freezes in space as the camera pans around it, showing it from different directions. (In fact what is happening is that a series of cameras at different angles capture the moment, and their images are linked together by a computer to produce the illusion that the camera is panning.) Imagine that we do this for real. We stop time at that one instant and view the arrow.

Now let's do the same for another arrow that isn't moving at all. We won't worry too much about how this second arrow is suspended in space. If it's really a problem for you, we could work the paradox with two trucks, one moving and one stationary, but Zeno used an arrow, so I'd like to stick with that. The question Zeno asks is: how do we tell the difference? How does the *arrow* tell the difference? How does the first arrow know that it must change positions in the next moment, while the other, seemingly identical in our snapshot, stays still?

It's a wonderful problem – where Achilles and the tortoise can seem almost a matter of semantics, this one is a real poser. In fact, arguably it wasn't possible to truly answer Zeno until another great thinker also imagined bringing something to a stop. We have to leap forward 2,400 years to find Albert Einstein lying on a grassy bank, letting the sunlight filter through his eyelashes.

Relaxing on a summer's day at a park in the Swiss city of Berne in the early 1900s, Einstein imagined freezing a beam of light, not by taking a snapshot in time but by riding alongside it at the same speed. Now, as far as Einstein was concerned, the light was stopped. This is just the same as if you were in a car, and a truck

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express

was alongside you going at exactly the same speed – from your viewpoint the truck would not be moving. But Einstein's daydream of stopping light was a real problem, because the mechanism that the Scottish physicist James Clerk Maxwell had used to explain the workings of light some fifty years before would not allow it.

Maxwell's explanation of light depended on electricity and magnetism supporting each other in a continuous dance, an interplay that could only work at one speed, the speed of light. If it were possible for light to slow down, the delicate interaction of electricity and magnetism would collapse and the light would cease to exist.

If Maxwell was right, and Einstein assumed that he was, light could only continue to exist if it travelled at that one speed. And so Einstein made the remarkable leap of thinking that light would always move at that one particular speed, however fast you moved towards it or alongside it. Where we normally add speeds together when we move towards another moving object, or take speeds away from each other when we travel in the same direction as something else, light is a special case that won't play the game. This idea is at the heart of special relativity, which then, conveniently, makes Zeno's arrow less of a problem. Because it soon became apparent to Einstein that fixing light to a single speed (around 300,000 kilometres a second) changes the apparent nature of reality.

Einstein combined this fixed speed of light with the basic equations of motion that had stood unchanged since Newton's time. He was able to show that being in motion would change the appearance of an object. Everyday, apparently fixed, properties such as size, mass and the passage of time itself would seem different for an observer and the object that was observed. This effect is not very obvious until you get close to the speed of light, but it is always there. Einstein was able to show that moving at *any* speed with respect to something else changed both how you looked to that observer and how the observer looked to you.

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express

Being in motion changes your world. And this provides a mechanism for the arrow to 'know' that it is in motion, because the world looks different in comparison with the world seen by a static arrow.

Relativity also deals with Zeno's final and least obvious paradox. This, called the stadium, imagines two rows of people passing each other in opposite directions. An athlete in one of these two rows will have passed twice as many body widths in the other row as he would have done if he had been running beside stationary people. Even though there is a physical limit to his speed, he seems to have run at twice that rate. In a way, what happens here is that our better understanding of relativity vindicates Zeno's views. He was using the example to show that the idea of moving at a certain speed is meaningless – and he was right. It is only ever possible to say we are moving at a certain speed relative to something else.

So with Zeno's four paradoxes in place, let's return to that sequence: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$ Is there a number of fractions we could add together that would bring the total up to 2? It would hardly be surprising if you said an infinite number. But what does that mean? If you ask children what infinity is when they are first introduced to it, they often say it is 'the biggest number that there can be'. Yet we've already said that, however many times you add on one of these decreasing fractions, you will never quite make 2. If infinity is 'the biggest number that there is', then surely we end up with a fraction that is

$$1 + \frac{\text{the biggest number} - 1}{\text{the biggest number}},$$

still not the round figure of 2.

This was a problem that proved an irritation for the ancient Greek philosophers who spent time thinking about such series. But to get into the frame of mind of a Zeno or Plato, we need to look at the numbers as the Greeks did themselves. The very wording of the sentence at the end of the previous paragraph

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express

would have been meaningless to the Greeks, because they did not have the same concept of a fraction that we do. In fact all round, numbers were handled in a very different way when the ancient Greek civilization flourished.

created with an unlicensed copy of nitro PDF express
upgrade: www.nitropdf.com/express